Beam-weapons defense is feasible: The great cyclotron controversy

by Robert Gallagher

Various physicists close to the Soviet-controlled Pugwash Conference on arms control such as Richard Garwin of IBM, Hans Bethe of Cornell University, and Kosta Tsipis of the Massachusetts Institute of Technology, have asserted that President Reagan's program to develop directed-energy beam technologies for anti-ballistic missile (ABM) systems is "impossible" or "not feasible."

Identical claims have been made in the past by theoretical physicists that were rapidly swept aside by scientific and technological achievements. An instructive example of such refutation of this "flat earth" school of physics is the course of a similar controversy over beam technologies touched off by Bethe in 1937, when he asserted that there was a "relativistic" limit to the energy to which circular accelerators, such as Ernest Lawrence's cyclotron, could accelerate ions—charged atomic particles such as hydrogen nuclei (protons).

The case of the cyclotron controversy is paradigmatic for policy makers, physicists, and others seeking to implement the Reagan directive. It demonstrates that the algebraic methods associated with theoretical physics can provide no basis for evaluating the feasibility of a beam-weapon ABM defense or for solving any remaining problems. Only geometrical methods based on the primacy of circular rotation are applicable to the study of relativistic and other high-energy-dense regimes. The cyclotron controversy proves this for anyone who cares to look. Relativistic phenomena express themselves in the visual universe through rotational motion or other harmonic orderings.

Bethe, on the other hand, has publicly rejected the geometrical method of the Leibnizian scientific tradition that provided the basis for all technological advances since the steam engine. In an interview published Dec. 3, 1979, Bethe himself complained to *New Yorker* magazine about the Ger-

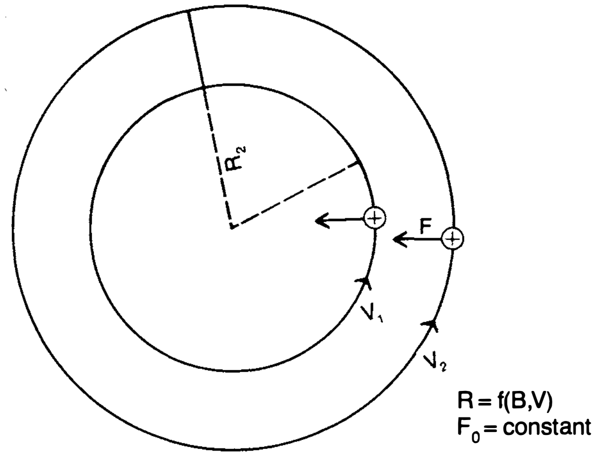


Figure 1. Rotational movement of charges between magnets. A charged particle moving between the poles of a magnet is turned to move in a circular path by a magnetic force F. With a constant magnetic force turning the particle, the greater its speed, the greater the radius of its circular path. In the figure, the ion with a greater speed, v_2, travels in a wider circular orbit. The magnetic force is directed inwards toward the center of the orbit. Lawrence realized that magnets could be used to set up "containers" of magnetic force within which to repeatedly accelerate ions to greater and greater speeds in a circular path.

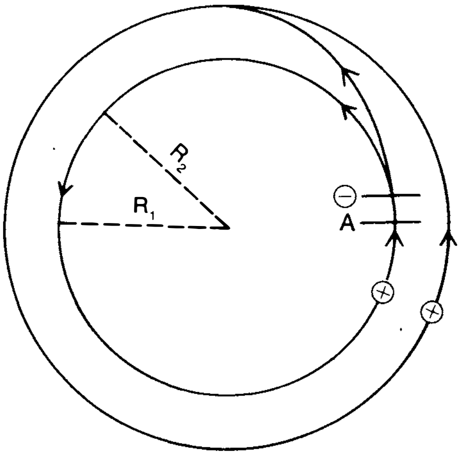


Figure 2. Voltages accelerate charged particles. A positively charged ion is accelerated by a negative electrical potential or voltage. In the figure, an ion is accelerated by the voltage at A to move at a greater speed in a wider circular orbit. The period, or duraction of one orbit, is the same as before acceleration. The number of orbits per second is the ion's orbital frequency, known as its "cyclotron frequency." This is constant if there is a constant magnetic flux between the poles of the magnet.

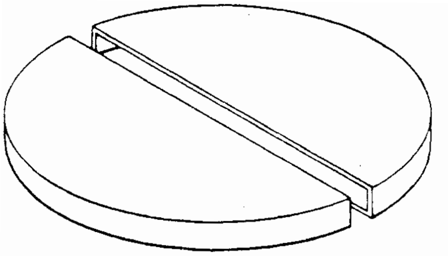


Figure 3. Semi-circular electrodes of the vacuum chamber of a cyclotron. The beam travels inside in a circular path.

man 1920s Gymnasium's geometry curriculum established by Felix Klein: "I wasn't particularly interested in geometry—ever. What I was interested in was algebra." It was Klein's emphasis on geometrical method that made possible modern aerodynamics that has since taken us into space.

Bethe had to wait nearly 30 years to receive the Nobel Prize for work he did in the 1930s on stellar fusion because his blunders on the cyclotron had damaged the credibility of his work.

Cyclotron harmonic resonance

Accelerators perform work upon beams of atomic particles by raising the beams' frequency of rotation as they propagate through space. All modern accelerators make use of the principle of harmonic resonance to perform this work. They are all harmonic oscillators.

In 1931, Ernest O. Lawrence and M. Stanley Livingston demonstrated the invention of the "cyclotron" circular particle accelerator with the generation of a beam of 1.2 million electron volt (MeV) protons. (One electron volt is the kinetic energy of an electron accelerated across a voltage of 1 volt.)

The design of the cyclotron was based on Leibniz's principle of least action: that the action of circular rotation is the only form of action that is self-evident in visible space. To understand the controversy touched off by Bethe it is necessary to grasp the simple physical principles that underly the operation of the cyclotron.

The cyclotron is based on the fact that a charged particle moving in a circular path under a constant magnetic field completes the same number of orbits per second regardless of its speed if that speed is not a large fraction of the speed of the light (see **Figure 1** and **Figure 2**). On this basis, Lawrence conceived of the cyclotron in which magnetic forces confine beams of particles to travel in circular paths in a squat circular vacuum chamber (see **Figure 3**). An accelerating voltage is applied across the gap between the two semi-circular halves of the accelerator chamber (see **Figure 4**). In order to repeatedly accelerate the beam, the cyclotron rotates the polarity of this voltage at the "cyclotron frequency" with which the beam orbits the circular chamber (see **Figure 5**). The rotation of the voltage is tuned to the cyclotron frequency. This condition is "resonance."

Machines developed prior to the cyclotron (such as electrostatic generators) accelerated particles *once* across a large voltage in a linear path. Applying the principle of least action through circular rotation, Lawrence accelerated ions to high speeds through repeated accelerations across a small voltage. The cyclotron is based on an elementary spiral work function.

Beams of accelerated ions from the cyclotron unlocked the atom and provided a means to study nuclear fission. By the late 1930s, under the direction of a cadre of physicists Lawrence trained, most principal physics labs in the United States and Europe had built cyclotrons as a tool to study radioactive disintegration of particles. Throughout the decade, Lawrence and his associates built a series of more powerful cyclotrons, achieving proton energies of 14 MeV by 1939.

The impact of cyclotron technology reached beyond physics. Lawrence and his brother John, a physician, founded a new branch of medicine by applying cyclotron ion beams to treat cancer. At first there was considerable resistance from the medical profession. But in 1938 the Lawrences cured their mother, aged 65, of terminal cancer with cyclotron ion beam therapy. She then lived to the age of 83.

In 1939, Lawrence received the Nobel Prize in physics for his invention.

Bethe's dissonance

In the midst of the acclaim over the benefits of Lawrence's discovery, Hans Bethe asserted that there were absolute limits to the operation of the cyclotron or any other

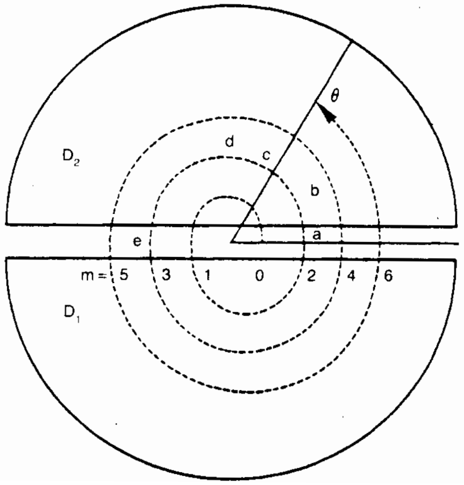


Figure 4. View of the cyclotron accelerator chamber from above. The accelerating voltage is applied across the gap between the two semi-circular electrodes of the chamber. To accelerate the particles, the polarity of the voltage must rotate (see Fig. 5) so that the electrodes (D_1 and D_2) are alternately negative with respect to each other during acceleration. For example, an ion passjing between the two halves of the chamber at "a" is accelerated by a negative voltage. D_2 is negative relative to D_1. By the time the particle again approaches the gap between the electrodes at "e", the polarity of the voltage has rotated: D_1 is negative relative to D_2 and the particle is again accelerated. By the time the particle has reached "4", the voltage has rotated again. As a result, the ion traces out successive semicircles in a spiral within the accelerator chamber. With each acceleration and gain in speed, it "jumps" up the magnetic potential to a wider circle. Notice that the path the beam takes between the gaps is always a semi-circle. If the voltage did not rotate, the particle would be alternately accelerated and decelerated.

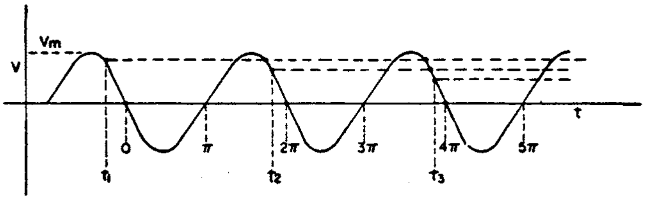


Figure 6. Loss of phase between the rotating voltage and the beam in the cyclotron. The sinosoidal wave represents the rotation of the voltage. Notice that in successive accelerations at t_1, t_2 and t_3, the beam trails the voltage peak more and more. Ultimately, it will cross the accelerating gap when the voltage is zero and as a result not be accelerated.

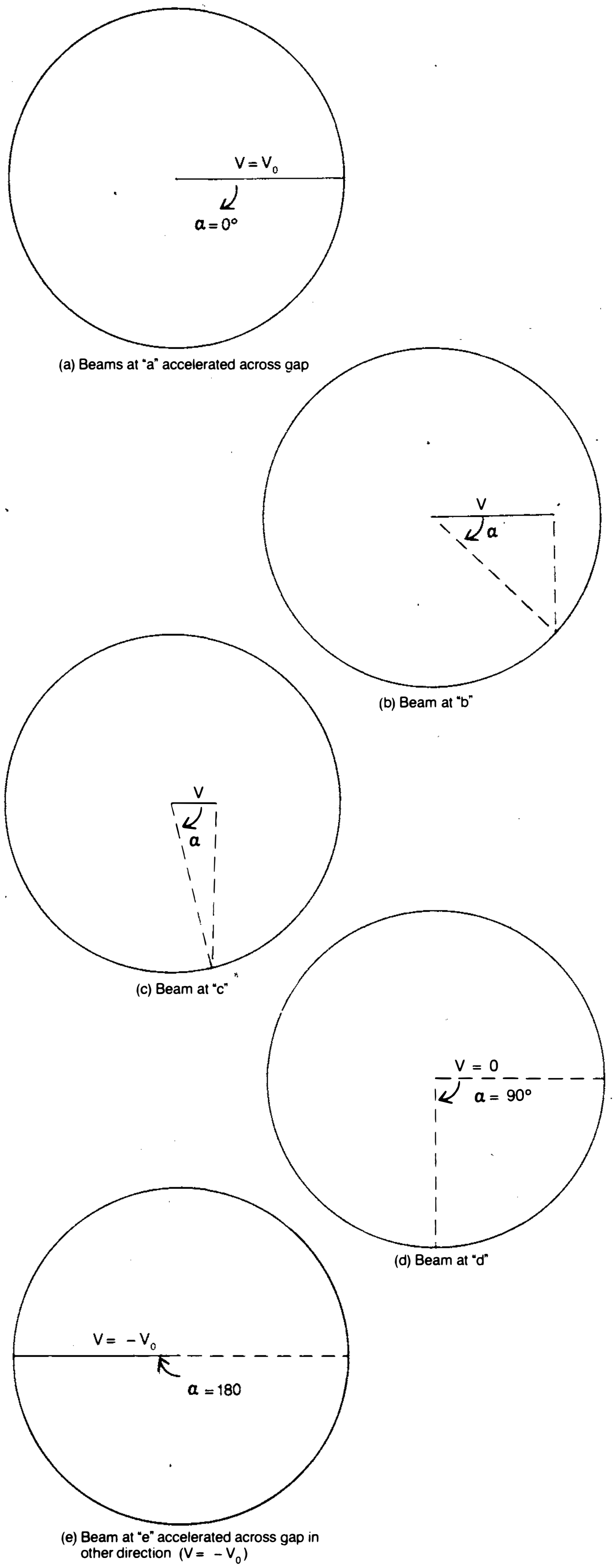
machine based on its principle of resonance. He expressed this thoughtless prophecy in a series of articles written with Morris Rose, an assistant, in *Physical Review* in 1937 and 1938.

Bethe's argument was as follows: At speeds that are a significant fraction of the speed of light (i.e., so-called "relativistic velocities"), the beam appears to slow down. When this occurs, the beam falls out of resonance with the rotating voltage that accelerates the beam (see **Figure 6**). The faster the particles are accelerated, the more they get out of phase with the rotation of the voltage, and the less and less the cyclotron accelerates the beam until finally it would no longer accelerate it at all. (This effect was well known by Lawrence and his associates.)

Bethe claimed that it was impossible to surpass this "limit," that there was no way to modify the cyclotron to restore resonance without defocusing and losing the beam. He expressed his results in the form of some mathematical fakery to "prove" that the problem could not be solved.

Bethe assumed that the only possible solution to the phase dilemma would be to increase the strength of the magnetic forces confining the beam as it swept out wider and wider circular orbits so that the beam would be pulled back into phase. This was already known to result in defocusing the beam (see **Figure 7**). Bethe's only contribution was to further document this negative result. He showed that increasing the

Rotation of voltage on cyclotron



(a) Beams at "a" accelerated across gap

(b) Beam at "b"

(c) Beam at "c"

(d) Beam at "d"

(e) Beam at "e" accelerated across gap in other direction ($V = -V_0$)

Figure 5. Rotation of accelerating voltage in the cyclotron. This sequence of figures shows the rotation of the polarity of the accelerating voltage through two accelerations of a particle in the case where the particle always crosses the accelerating gap when the voltage is maximum. Figure 5a shows the voltage at maximum V_0 as it accelerates the beam across the gap at "a" in Fig. 4. As the beam turns around the circular chamber, the voltage rotates so that its polarity is reversed and has a maximum value when the beam reaches "e"—accelerating the beam again as a result. Figs. 5b, c and d show the rotation of the voltage and the decline in its amplitude to zero while the beam passes "b", "c" and "d" in Fig. 4. Fig. 5e shows the voltage rotated 180 degrees as the beam crosses the gap at "e". The voltage is always its maximum V_0 times the cosine of the angle of rotation, α. The rate of rotation of the voltage is tuned to the orbital or "cyclotron frequency" of the beam. In future accelerators, the voltage rotated at an octave or harmonic of the cyclotron frequency.

magnetic forces at the periphery of the chamber by only one percent would result in exponential spreading of the beam and its reduction to one billioneth of its former intensity. In the cyclotron, to maintain a focused beam, the intensity of confining magnetic forces is made to decrease with radius.

For purposes of documentation, we quote from his *Physical Review* articles:

"We see that either the resonance or the focusing is destroyed by the relativistic change of mass irrespective of the special choice of the magnetic field. . . . Thus it appears that the cyclotron cannot be made to give much higher energies than those obtained thus far. . . . Various possibilities for improving either the focusing or the resonance by changing the design of the cyclotron were considered but there are objections to each of them." For cyclotrons with a maximum accelerating voltage of 50,000 volts, Bethe and Rose asserted that "the maximum attainable energies" were 15.6 MeV for protons, 22.15 MeV for deuterons and 44.3 MeV for alpha particles. British Nobel Laureate physicist James Chadwick, the discoverer of the neutron, echoed Bethe's argument in a 1938 *Nature* article.

Are 'relativistic effects' limiting?

Bethe's assertion was that the harmonic laws upon which the cyclotron was based were valid only at low energies, that the harmony of the universe—demonstrated successively by Plato, Nicholas of Cusa, Kepler, Leibniz, and Carnot—broke down as the beam approached the speed of light.

In fact, the opposite was the case: As a particle beam becomes relativistic, it *characteristically* exhibits additional harmonic motions.

Commenting on the algebraic method characteristic of Bethe, *EIR* founder Lyndon LaRouche has recently stated: "It is a fool's enterprise to attempt to wring out of such mathematics any evidence bearing upon causation: One would have better luck attempting to wring blood from a stone."

American and Soviet physicists developed several solutions to the limits enunciated by Bethe. They were based on the discovery that the accelerating beams develop two additional forms of harmonic motion and so exhibited self-organizing processes. Methodologically, the solutions rested on the work of Leibniz and Carnot.

Leibniz's fundamental principles of physical geometry were the principle of continuity, the principle of conservation of *vis viva*, and the principle of least action. Carnot's 1783 *Essai sur les machines en genéral* and other writings provided their elaboration in his theory of machines.

The principle of continuity states that matter is characterized by self-organizing processes and that such processes are the key to identifying the continuous path of physical transformation between two qualitatively distinct physical regimes, such as "relativistic" and "non-relativistic."

Virtual motions

To address the conceptual problems of engineers and physicists in applying this principle to machine design, Carnot developed the concepts of "virtual" and "geometric" motions based on Leibniz' principles. Leibniz defined "virtuality" as "the momentary transition determined by force and the necessity of its change." Otherwise, "there is nothing real in motion itself." In other words, work brought forth "virtuality" in matter, its tendency towards change.

Carnot applied this understanding of the self-organizing tendency in matter to machines. He wrote that the virtual motion of any system of bodies (such as a particle beam) is that which it would exhibit at any moment if it were no longer constrained (for example, by the forces exerted by the accelerator), "if it were free," he stated. His criticism of 18th century engineers was that they did not design machines in accordance with the fact of virtuality. As a result, contemporary machines quickly reached "limits" or *singular states* in their action because their action was in part directed against the virtual motions they brought forth in matter.

Carnot emphasized that for maximum power, machines must apply forces or execute motions that were consistent with virtual motions. Such motions are "geometric." They

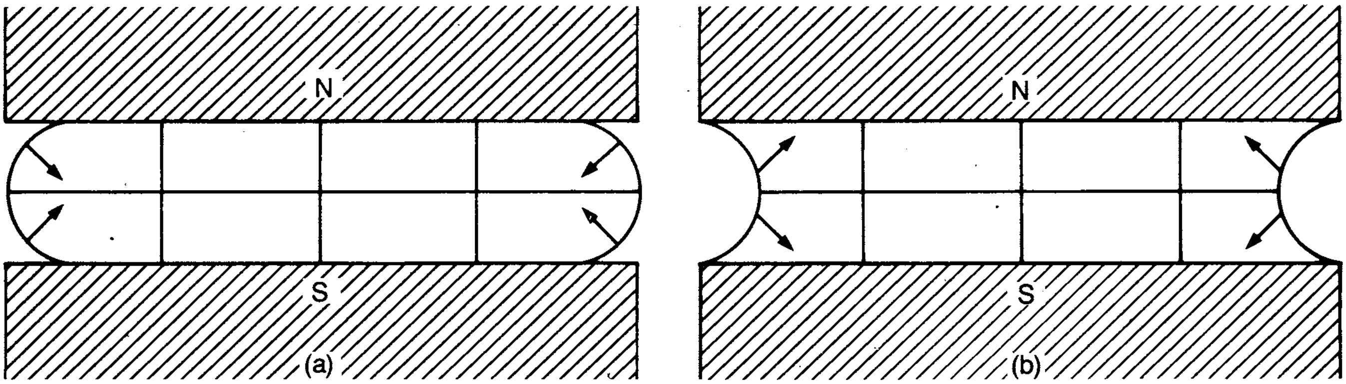


Figure 7. Focusing and defocusing of the beam in the cyclotron. The figures show a vertical section of the cyclotron magnets and the accelerator chamber between them. If the strength of the magnetic force declines slightly with radius, the beam is focused towards the median plane of the accelerator chamber. Fig. 7a shows that the magnetic field lines are convex at the periphery of the chamber in this case and the focusing forces always directed towards the mid-plane. Fig. 7b shows the concave shape of the field at the periphery and the direction of the focusing forces—if the magnetic field increases slightly with radius. The beam is always defocused. Bethe argued that this effect made it impossible to restore resonance in the cyclotron or any machine based on its principle of resonance.

would not act against virtual motions, and essentially established the boundary conditions under which virtual motions would evolve. Geometric motions would not "change the reciprocal action of the different parts of the system"—as Carnot stated in his 1803 *Principes fondamentaux de l'equilibre et du mouvement*—so that under application of geometric motions, the system can evolve *continuously* from one state to another. As he elaborated:

> Any motion that, when imparted to a system of bodies, has no effect on the intensity of the actions that they exert or can exert on each other in the course of any other motions imparted to them, will be named geometric.

The key to machine design is to identify the geometric motions that will carry the machine past singular limits of action that would otherwise appear. Carnot emphasized that one distinct physical regime or machine state is the projection of a preceding one via a projective transformation carried out with geometric motions. Furthermore, he stressed that efficient action was always quantized and that in continuous-process machines, the action must change only by quanta or "by insensible degrees," as he wrote, since this would be consistent with the evolution of virtual motion.

Carnot demonstrated to the engineering community of Europe that machines designed with such a geometric method would exhibit conservation of *vis viva*: They conserve the self-organizing processes exhibited as virtual motion. Such processes are formally termed "adiabatic." Carnot's son, Sadi, applied his work to develop a theory of the adiabatic operation of heat-powered machines.

Harmonic 'self-focusing'

At the end of World War II, two physicists, V. Veksler of the Lebeder Physical Institute in Russia, and E. McMillan, a collaborator of Lawrence, both simultaneously published solutions to Bethe's difficulty, solutions based on the fact that the beams exhibit "self-focusing." Veksler dubbed his accelerator designs "adiabatic" after Carnot.

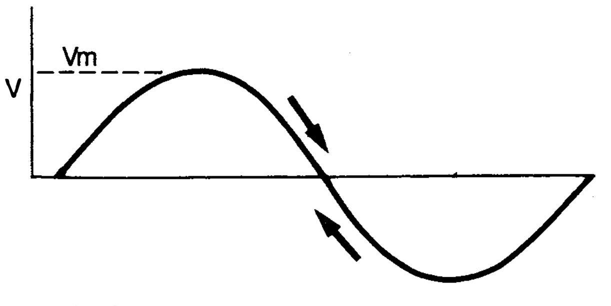


Figure 8. "Self-focusing" of the beam in the cyclotron. Veksler and McMillan showed that loss of phase would result in alternate acceleration and deceleration of the beam so that its phase with the rotating voltage would swing back and forth about a phase with V = 0. To continuously accelerate the beam it is only necessary to increase the intensity of the magnetic flux in small steps so that the beam's phase swings about a positive voltage and is always accelerated. (Modified from Ref. 4.)

Contrary to Bethe, Veksler showed that the loss of resonance was predictable, that during each circuit of the beam within the cyclotron, it went out of phase with the rotation of the accelerating voltage by a quanta of virtual action dependent on the maximum accelerating voltage and the magnetic flux intensity. This phase rotation would stop when the phase finally coincided with zero voltage at the accelerating gap so that there was no net acceleration. McMillan further demonstrated that when resonance was thus "lost," the beam simply swung between accelerating and decelerating phases *with the harmonic motion of a pendulum* (see **Figure 8**).

The required geometric motion to return the beam to phase was simply to give it a small kick with each orbit by stepping up the intensity of the magnetic forces confining the beam so as to rotate the beam back to an accelerating phase with the voltage. The end result of these geometric motions was that the beam always swung about an accelerating phase. McMillan represented this geometric approach with a model (see **Figure 9**). He and Veksler showed that step-modulation of the frequency of rotation of the voltage would also serve as a geometric motion to restore resonance.

The quantum of least action

Veksler emphasized that the critical element with either magnetic or electric force variation was for the steps or "quanta" to be "sufficiently small compared with the total field," or as Carnot argued, the variation must be "by insensible degrees." Beam-focusing is maintained as in the cyclotron with magnetic forces that always decrease with radius even though they increase with time. Two of McMillan's associates wrote, "The apparent difficulty [of loss of phase] is converted to an aide, allowing a theoretically unlimited number of successive accelerations." The singularity that arose in the conventional cyclotron was surpassed.

Immediately, McMillan introduced variation in the frequency of the accelerating voltage into the 184-inch diameter Berkeley cyclotron. The machine—renamed the synchrocyclotron—achieved energies of 200 MeV for deuterons applying a maximum accelerating voltage of 15,000 volts, less than one-third of that Bethe asserted would be required to achieve a "maximum" energy of 22 MeV.

In 1952 the Berkeley proton synchrotron—based on variation in both the magnetic forces and the rate of rotation of the accelerating voltage—produced protons of 6.2 billion electron volts (GeV). The machines demonstrated that—contrary to Bethe's lattice of algebraic constructs—relativistic regimes of the universe are harmonic.

Variation of the magnetic forces permitted the use of a donut-shaped accelerator vacuum chamber (see **Figure 10**) since the beam can be kept orbiting at a nearly constant radius.

Does mass increase with velocity?

The limits of the synchrotron were known before the first machine was built: Because iron magnets saturated at about 1800 Gauss, there was a limit on magnetic field intensity; higher energies could only be obtained by increasing the radius of the toroidal accelerator chamber. Then American physicists designed machines around the second additional harmonic "virtual motion" that the beam exhibited only at relativistic speeds. To do so required subjecting one of the central tenets of Einstein to criticism.

D. W. Kerst first demonstrated in his circular Betatron electron accelerator that beams of accelerating particles spiral (rotate) about their mean circular path in the machine. (See **Figure 10**). This movement was named "Betatron oscillations." They are the principle geometric correlate to higher beam energies.

But the magnets of the synchrotron were designed on the assumption that the least action path for the ions was a simple circle, not a spiral path. As a result, the magnets "fought" against the natural spiral motion of the beam. Accelerators that would surpass the limits of the synchrotron must accelerate the beam along its "natural" spiral trajectory.

The fact of Betatron oscillations explain why accelerating beams took longer to make one orbit of the cyclotron as their energy increased. As the machine accelerated a beam to higher speeds, the rate of these Betatron oscillations increased. As a result, the path that the beam travels in the machine lengthens (see **Figure 10**). Its period of orbit *must* increase!

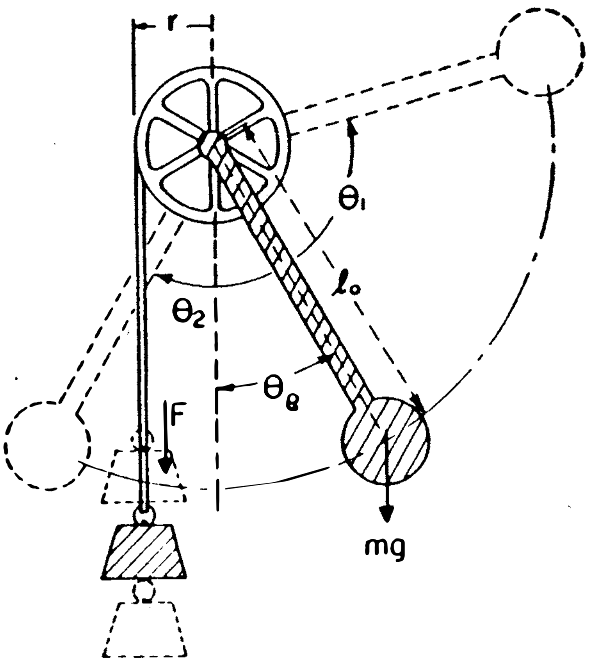


Figure 9. Geometric motion in the synchrotron. The diagram shows the pendulum model of the rotation of the phase between the beam and the accelerating voltage in the synchrotron. The weight that prejudices the movement of the pendulum to the right represents the geometric action of increasing the magnetic field strength in quanta to keep the beam at an accelerating phase with the voltage. In the cyclotron, there is no such geometric action applied so that at relativistic velocities, the pendular motion of the phase is about a voltage of zero, or no net acceleration. (Ref. 4).

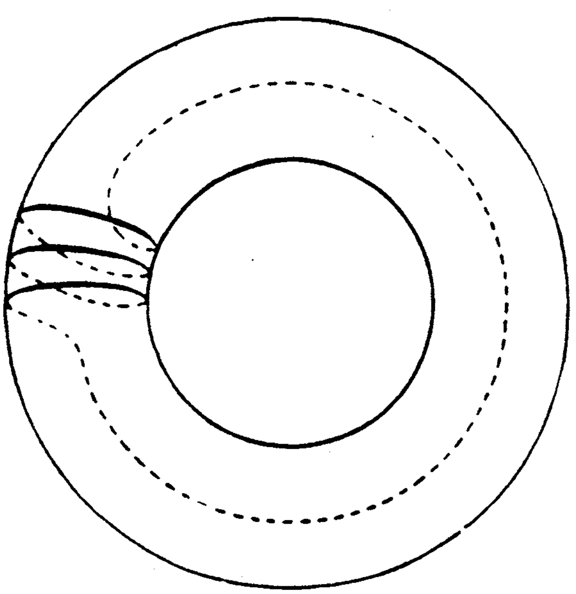


Figure 10. Betatron oscillations are a spiral rotation (solid lines) about the mean circular path (dotted line) that a beam takes around the donut-shaped accelerator chamber shown above.

This geometric explanation of a "relativistic effect" showed that the popular fiction that mass (or inertia) increased with speed (velocity) is meaningless. If this doctrine was to have any value, it would form a basis for problem-solving. It does not. Because it reduces relativistic effects to the point, the particle, the notion that mass increases with velocity can only justify the view that the problem of increasing beam energy in the cyclotron was unsolvable. Bethe used precisely this approach.

In fact, the apparent increase in inertia of the beam was based only on its evolution into a new geometric form, its (Lorentzian) contraction into a coil or spiral. The beam increasingly behaved as a wave, not as a mass of particles. Because it contracted with every orbit, it lost phase with the cyclotron.

The path of least action

In 1950, Nicholas Christofilos, an engineer who designed accelerators as a hobby, proposed a magnet system for a synchrotron that would accelerate particles along a spiral trajectory in an application to the U.S. Patent Office. The system would achieve synchrotron energies with "smaller magnet weight and smaller cost." Christofilos based his theoretical treatment on Bernhard Riemann's harmonic function theory. His patent application was immediately classified.

In the magnet system proposed, accelerated particles would be subjected to harmonically rotating magnetic forces all along their path. The action ot these forces produces motions that are geometric relative to the beam's virtual spiral rotations.

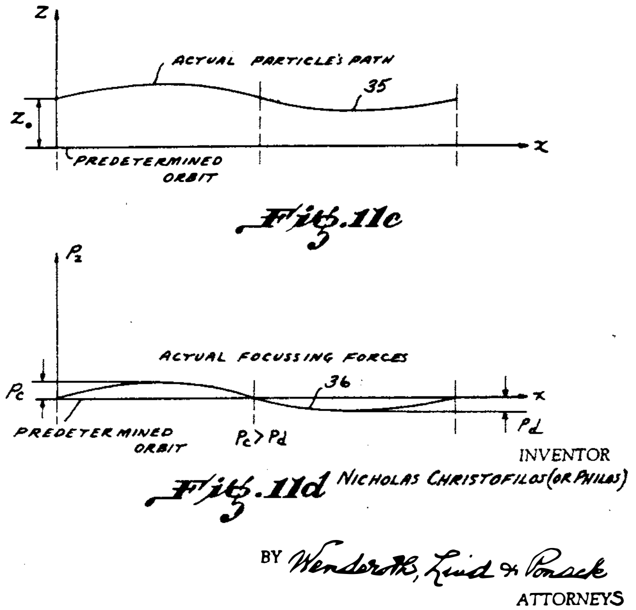


Figure 11. Geometric motion in the alternating gradient synchrotron. These figures are from Christofilos' original patent application. Fig. 11c shows the resulting spiral path of the beam; 11d shows the rotational magnetic force applied, geometric relative to the (virtual) spiral rotation of the beam (Ref. 3).

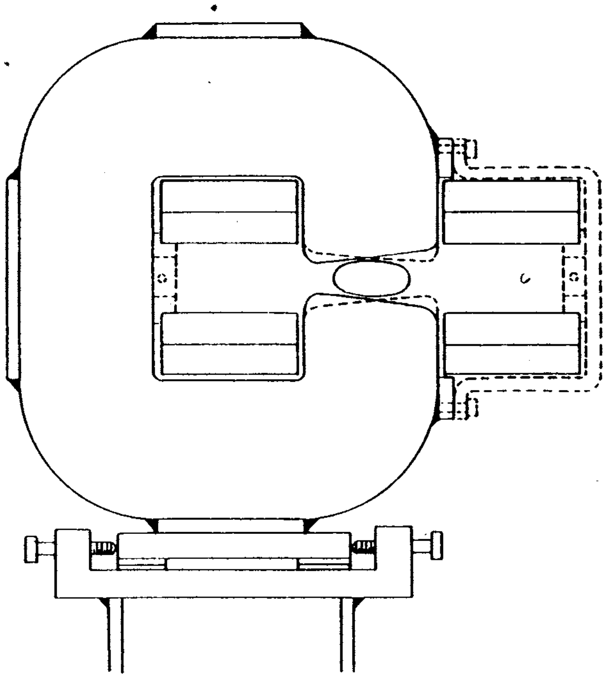


Figure 12. Magnet from an alternating gradient synchrotron. The magnet poles form the sides of a rectangular hyperbola. The dotted lines indicate the poles of the other member of an alternating gradient magnet pair in which the orientation of the poles is reversed. If the North Pole is above the South in the first magnet, it is below in the second of a pair. Note the elliptical cross-section of the accelerator chamber between the magnet poles. (Ref. 3).

Figure 11 from Christofilos' original patent application shows the rotating magnetic forces and the resulting spiral movement of the beam in his design. In the new machine, the rotating forces are produced by magnets whose poles form sides of a rectangular hyperbola and whose poles alternated in orientation along the toroidal chamber of the accelerator (see **Figure 12**).

Christofilos' design for a quadrapole magnetic lens from his 1950 patent is employed in the radio frequency quadrapole accelerators that form the basis of the particle beam weapons programs in the United States and Russia.

Stanley Livington and Ernest Courant independently discovered the principle of Christofilos' design and built an accelerator based on it at Brookhaven. It achieved 32 GeV protons in 1960.

Less quarkery, more geometry

The attainment of higher energies is not an end in itself. The importance of the breakthroughs described here is that they show that development of smaller, relativistic particle accelerators for beam weapons is feasible. The application of the principle of least action produced accelerators that achieved successively higher particle velocities and currents with lower accelerating voltages and lower magnet weight per length of the accelerating chamber. Table I presents this historical fact. Our goal is to apply the least action principle to develop relativistic particle accelerators that you can fit in a tank.

The first step toward this goal will be for physicists to abandon the cults of algebraic analysis and "high energy physics." High energy physics is a fraud. Its study of subatomic particle interactions has become a form of voodoo: It assumes the ontological primacy of the point, the particle, to the rotational motion that is primary.

Today, most physicists regard Betatron oscillations as some form of "instability" or "experimental artifact," rather than a characteristic of particle beams. The situation is similar to that which exists in the U.S. magnetic fusion energy program where practitioners are applying the brute force of strong magnets to confine a plasma, rather than designing machines to bring out the "virtuality," or self-organizing tendency of

Table 1
Least action parameters of accelerators

Machine	**Max. Voltage** (kilovolts)	**Magnet. flux** (Gauss)	**Mag. wt. per meter** (tons)	**Max. "energy"** (MeV)
Cyclotron (Berkeley)	100	15700	25	8.5 d+
Proton synchrotron (Brookhaven)	1	14000	29	3,000 p+
A-G synchrotron (Brookhaven)	80	13000	4.5	30,000 p+

Note: d+ = deuteron beam, p+ = proton beam.

the plasma to perform work upon itself.

The simplest geometrical constructions provide more guidance for mastering "relativistic effects" than Einstein's mass-energy doctrine. The cone represents the self-similar rotational action fundamental to the universe and characteristic of a work function that *progresses from non-relativistic to relativistic regimes*. A simple comparison of the cyclotron to the synchrotron illustrates this.

In the naive view, geometry is comprised of the study of three spatial and one time dimension. Under this prejudice, physicists view processes as dynamical interactions of particles and forces in visual space. Such a view was the basis of Bethe's outlook. In first approximation, the actual dimensions of physical geometry are not mere spatial coordinates, but the degrees of variation in the physical processes themselves.

In any cyclotron, the magnetic forces confining the beam are constant, the maximum beam energy is determined by how far out the beam can spiral, that is, by the mere size of the vacuum chamber. The cyclotron frequency, or rate of acceleration is also constant. The action of the cyclotron is limited.

The breakthroughs of Veksler, McMillan, and Christofilos in developing the synchrotron permitted a dramatic increase in the ability of an accelerator to perform work upon the beam. The principal degrees of freedom (leverage) in the geometry (or phase space) of action of the relativistic synchrotron do not exist in the cyclotron:

1) The rate of increase of magnetic forces;

2) The (related) increasing rate of rotation of the accelerating voltage (the increasing rate of beam acceleration); and

3) The rate of rotation of magnetic forces.

In the Brookhaven Labs' proton synchrotron, magnetic force strength increases 50-fold over the course of the acceleration as the number of accelerations per second, the beam's now variable "cyclotron frequency," increase 10-fold. Increasing the strength of magnetic forces in small steps or quanta corresponds to a gradual expansion of the size of the magnetic "container" to accommodate the increasing energy of the beam—unlike in the cyclotron where the size of the container is fixed by the constant magnetic force applied.

In the Brookhaven alternating gradient synchrotron, the rate of rotation of the beam (Betatron oscillations) is an order of magnitude greater than in the protron synchrotron or cyclotron.

The geometry (or phase space) of the cyclotron is a cylinder; its action is limited. The geometry of the synchrotron is that of the self-similar expansion of the cone. The expansion of the cone represents the increasing frequency of rotation of the beam under the expansion of the "container" of magnetic forces. Accelerating voltages boost the beam to a higher (Betatron) frequency of rotation up the cone as this "container" expands. This action is represented by an elliptical cut through the cone designating the "jump" from one energy circle to another one (see **Figure 13**). Were it true that the physical geometry of a relativistic beam is conical, we would see this projected in some way into visual space. We do. The cross-section of a particle beam's Betatron oscillations in the alternating gradient synchrotron of Christofilos is an ellipse.

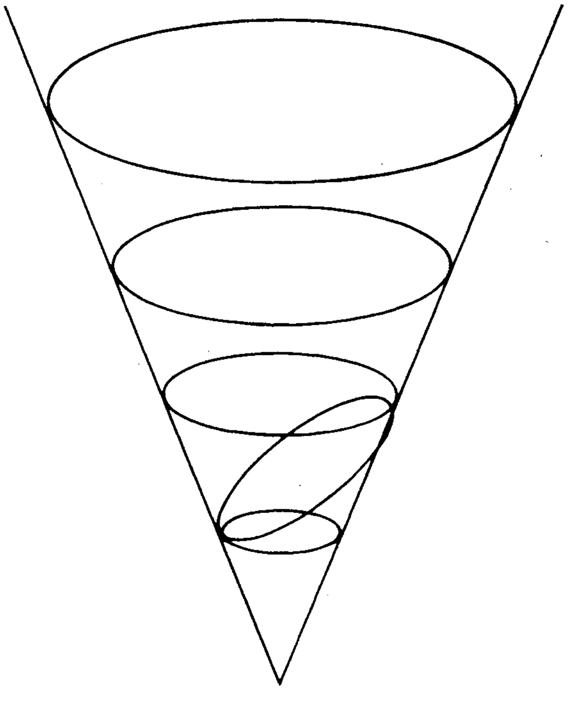


Figure 13. Conical model of the proton synchrotron. The cone represents the characteristic geometry of a (self-similar) work function effective in both non-relativistic and relativistic regimes. A quantum modulation of the strength and the rate of rotation of magnetic forces and of the rate of beam acceleration boosts it to a higher rate of "Betatron oscillations", or energy. This action is represented by an ellipse and the work performed by the volume between two such successive "energy" circles.

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