Euler's fallacies on the subjects of infinite divisibility and Leibniz's monads

Leonhard Euler (1707-83), renowned Swiss mathematician, astronomer, and natural scientist, studied mathematics for 11 years under Jean Bernoulli. Bernoulli had collaborated with Gottfried Leibniz, the German philosopher, statesman, and universal genius who invented the calculus, on various problems of mathematics and physics. But, in his 1761 Letters to a German Princess, *Euler attacks the followers of Leibniz, who had died 45 years earlier, in a manner revealing his own lack of understanding of Leibniz's notions of space, time, and substance.*

He was an opponent of the Newtonian reductionist method in mathematical physics. In an attempt to refute Newton's bowdlerization of Kepler's great discoveries, Euler tried to show that Newton's theory did not correctly account for perturbations of the Moon. While Euler was absolutely correct philosophically in his criticism of Newton's axiomatic barbarism, this could not be demonstrated for the case of the Moon's orbit.

LaRouche, in a three-part essay dictated by telephone from prison in the third week of January 1990, demonstrates the fallacies in Euler's argument and revives the standpoint of Leibniz's Monadology. *Following LaRouche's critique, we publish two of Euler's letters, which present the essentials of his argument.*

A critique by LaRouche

Let me deal first with the core argument by which means Euler introduces the subject (I'll deal later with the second part of his argument, which is more specific, on the subject of monads).

Euler obviously starts with a very simple proposition, winds up to it, then gets into monads, and premises the entire discussion which ensues on a certain fallacy. I shall now just summarily address that fallacy, specifically because it is very interesting to do so, as well as profitable.

He argues simply for the case of infinite divisibility, and I need not replicate his argument; it is clear enough. Simply by asserting infinite divisibility, he comes up against a problem which he ignores, a problem which was recognized implicitly as early as Leonardo da Vinci, in respect to physics *qua* physics.

All through the discussion of this subject, there's been the question: If we divide all observation into three categories, can we attribute the same sensory properties of phenomena to all three categories in the same fashion, without some qualification as we move from one to another?

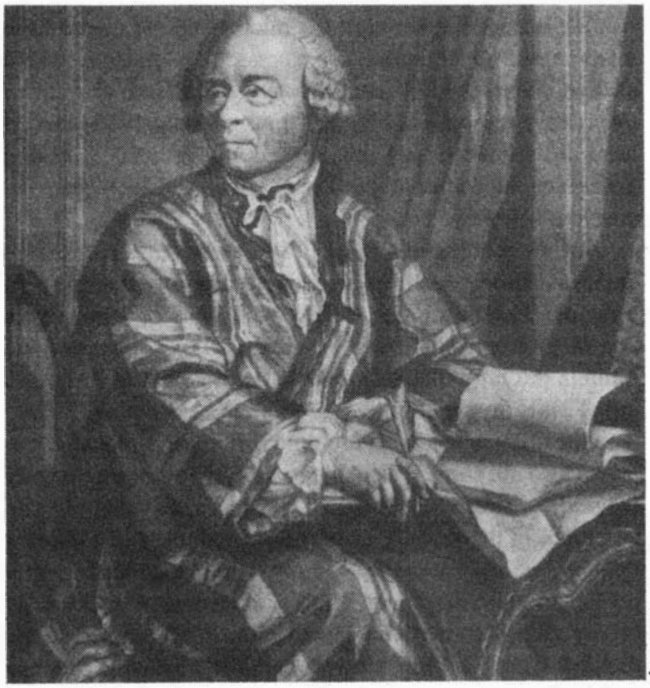
The three categories are the following:

First is the level of simple visual observation, simple sensory observation, a physical space-time as it appears to our senses by virtue of the limitations of our senses. The second is astrophysics, the macro-scale, that which is accessible in a sense to our senses, but which involves things which are far beyond our senses' immediacy. The third, of course, is microphysics, that which is so small, that it is beyond the capacity of direct observation by means of the senses.

Now, from early times up through Riemann, those of my persuasion have insisted that when we come to the extremes of astrophysics and microphysics, we can no longer make the simple projections which might be suggested by observation or successful observation within the realm of visible and kindred phenomena, on that scale.

This begs a third question: What is the nature of the boundary separating each of the extremes, i.e., the large, astrophysics, and the very small, microphysics, from the ordinary scale of observation.

Generally I think we accept the notion, or those of us do who ponder this matter, that we speak of microphysics as that which lies in the vicinity of such a boundary, as in microphysics, the very small. You might say an Angstrom unit, or two or three Angstrom units, might not be that boundary or might be that boundary, but that when you get down into micron and similar kinds of areas of measure, you are in a troublesome area, relative to projections simply of the ordinary rules of visible observation and visible phenomena. Similarly, when we deal with matters on an astro-scale or astrophysical scale, for various reasons, having to do largely with time and so forth, we can no longer trust the simple rules of observation, of visible related phenomena. So, we are not concerned, generally, when we speak of astrophysics or microphysics, with knowing, at least for preliminary purposes, the exact boundary which separates the classes of phenomena. But we say, "When we get in the vicinity of those, a certain area, a certain scale, we have to be alert for



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The Swiss mathematician Leonhard Euler (1707-83), revealed his lack of understanding of Leibniz's notions of space, time, and substance in the letters cited here.

sudden changes, abrupt changes hitting us."

We would say the boundary, of course from the standpoint of physics, is not a wall, but is rather a singularity. An example of that would be satisfying, since this was already addressed by Leonardo da Vinci in respect to sound, for example, and light. When we project a body under power to a supersonic speed, velocity, that it is not in this case impossible to have supersonic velocities, but certain changes occur within the realm in which this occurs, the transonic, supersonic phenomena occur, changes associated with phenomena which are not otherwise evident on the scale of observation of events at the lower speeds. So that the speed of sound is a singularity. A transonic area is a singularity, such that we cannot generalize what appears to be adequate interpretation of phenomena at lesser speeds as we move through the transonic to the higher speeds.

So that's what we mean, generally, when we say a change in the rules for observation of physical space-time as we encounter a boundary condition in the form of a singularity, as we continue to venture into the ever-smaller and the ever-larger scale.

The way we generally would approach this, particularly in the present century, is in respect to the limiting factor of the speed of light. As we approach the speed of light, we speak of a boundary area, which we call relativistic conditions. Generally, this is applied to the scale of astrophysics. But, ingenious minds will promptly attempt to reflect what is true of astrophysics, even as a consideration, back onto microphysics. That is, it is the common tendency in mathematical physics to treat the infinitesimal as an inverse of the infinite. Thus, if the speed of light is a boundary condition in the one scale, we must expect that there is a complementary boundary condition, i.e., a singularity in the microphysical scale. That is essentially the way this should be approached.

What this would mean, of course, is that there is no infinite divisibility, in the sense I just implied. That is, we are not talking about an impossibility of some kind of divisibility on the microphysical scale below the scale of this boundary, this singularity, but we are implying the singularity as such.

This whole business, in both instances, is associated with the issue of the proper definition of physical space-time itself. Is physical space-time, in respect to physical cause and effect, a matter of simple linear extension, or is it not?

Kepler's astrophysics says it is not a matter of simple linear extension: that the available planetary orbits are not only limited in number, in the sense of being enumerable, but that this enumerability is defined by a very definite, intelligible principle, a principle susceptible of intelligible representation, which is the harmonic ordering; and that in the values of a special kind of Diophantine equations, if you like, in the values which lie between these harmonically ordered, enumerable values, there are no states of a similar nature, or precisely similar nature, at least, to be found.

Now, this introduces a kind of discreteness into physical space-time per se. That physical discreteness is the first aspect of a monad in the microscale.

Let me skip a bit, and go ahead to another consideration respecting both astrophysics and microphysics. What about the large monads? The very large monads belong, not necessarily, immediately, to the microphysical scale, but rather to the astrophysical scale. Ahaaa! Right? Now there is a second consideration.

This goes to what I treated under the title of the Parmenides paradox: the immediate relationship between the infinitesimal and infinite, say in the case of a human being. In this case you will see that it leads to the second point, on the monad.

We in a sense are, in the scale of astrophysics, an infinitesimal. Our mortality makes us all the more so. Nonetheless, we can affect the universe as a whole, at least implicitly so. We do so by an agency; that agency is creative reasoning.

We are capable of discovering, less imperfectly, the laws of the universe, and doing this by creative reason. By activating and acting upon those discoveries by means of the agency of creative reasoning, that is, by acting on them by means of creative reasoning as well as discovering them by that means, we are able to influence the course of behavior of society as a whole, and society as a whole is able to act on the universe, on an ever-larger, implicit scale of chains of cause and effect. By that agency, in terms of discovering universal principles, less imperfectly, and by discovering more powerful and more efficient means of acting upon the universe in the large by these means, we show that the human individual, this mortal ephemeral creature, we, the individual, actually have an implicitly direct relationship to the universe at large.

Similarly, we come to the second principle. Not only is the monad, so-called, something which is defined in respect to scale, but it is defined in respect to an active principle. Now here we come to the crucial matter, as treated by Leonardo da Vinci, and treated explicitly by Kepler, as in the small paper *On the Six-Cornered Snowflake*.

On the ordinary macroscale of observation, it appears to us that we have two harmonic orderings: one characteristic of living processes, and the other characteristic of non-living processes, as Kepler treats this matter in *The Snowflake*. Thus, is the universe bifurcated in this way, or do we find some reflection of this question in the microphysical and macrophysical, or astrophysical, scale which removes the apparent paradox, or which makes comprehensible the apparent anomaly of the division of visible space-time and physical phenomena of observation into these two, living and non-living parts?

We find it just so. We find it implicitly required, for example, that the monads, in the scale of the small, in the microphysical scale, be implicitly negentropic, rather than entropic. That is, since negentropy, as a phenomenon, is characteristic of living processes, and entropy of non-living processes, then we must find, what might be considered by some, the simplest aspect of the non-living, the simple physical monad, to be implicitly negentropic—that is capable of showing negentropy or entropy, but being primarily negentropic. This again bears upon our relationship to the universe as a whole through creative reason, that is, our individual relationship to the universe as a whole as creative reason.

This goes to the simple *Parmenides* paper, to that little, beautiful irony, which is the center of that artistic composition, rightly called artistic. Amid all of these antinomies, this elaborate, quasi-deductive array of antinomies, Plato inserts a touch of irony: that after all, the problem here is that the transition between these qualities which seem paradoxical, is defined by change, and if we introduce, implicitly—Plato says, not explicitly, but implicitly—if we introduce change as having the primary ontological actuality, in this case, then the mystery of the antinomies dissolves and vanishes.

The problem here, is that when we say that this divisibility of physical space-time in its linear aspect is elementary, we get into precisely the problem which Euler creates here. So, by assuming that simple extension in that sense is the property of matter, we create all the chimera which haunt Euler's dream in this instance.

We recognize the implications of the speed of light as a singularity of the astrophysical scale, and recognize that the speed of light has a reflection in terms of a singularity in the microphysical scale, then we see where the fallacy of Euler's argument lies respecting physical geometry. If we recognize that the connection between the micro- and the macro-, the maxima and the minima, is expressed by change, where change is the quality of negentropy generalized, as typified by creative reason—as I have, I think adequately, defined at least in the preliminary degree, in *In Defense of Common Sense* and locations to the same effect, earlier—then the problem vanishes.

So, the problem for Euler lies in his definition of extension, and in the use of a linear definition of extension. In principle, Euler excludes, thereby, the realm of astrophysics and of microphysics from physical reality. This is where Leibniz did *not* fail, and where Euler, at least in this case, did. That is my preliminary observation.

One thing added, as a footnote: Microphysics and astrophysics do not simply stand independently of the universe of the scale of simple observation; but, there is a point of scale at which, in the vicinity of whatever boundary condition is defined, we must *change*. We must recognize that we can no longer rely simply on simpler elementary methods of observation, but must change our view to accommodate the fact that we are approaching a singularity. Thus, in practice and in fact, as we get into the very small, divisibility of the ordinary sense *vanishes*, as it does as we get into the astrophysical scale, where the relativistic considerations remind us, or should remind us, that we are approaching a boundary condition in that respect.

Thus, as we get to certain areas of scale, in practice we no longer trust infinite divisibility. What that exact boundary condition might be, as, say, from the standpoint of the eighteenth century, we might not know. But we must know that one does exist, as Leibniz recognized. We must also recognize, as Leibniz recognized and Euler *does not*, that there is a qualitative change in the immediate implications of phenomena, of existence, as we get into the microphysical scale, i.e., that that which seems to be entropic non-living processes, on the scale of simple observations, can no longer be treated as simply entropic, but as a negentropic existence susceptible of generating ostensibly entropic phase spaces.

Not only is Euler wrong—and it is important to find Euler wrong, because of how otherwise useful he is—but, I think he has made what we might call a *strong* error, which has tremendous pedagogical value.

Letter 12, on the subject of monads

I address the content, in part, of Letter 12 of Euler's letters on the same subject of monads.

Euler introduces a fallacious argument of some significance, an argument whose foundation is a simplistic reading of the *Monadology* by some critics of Leibniz's work. This pertains to the magnitude of monads. Are they greater or lesser? Since they cannot be greater or lesser by the method which Euler imputes, then the whole thing is absurd. He also, therefore, says that relative to magnitude, they are absolute nothings.

It is interesting to look at this from the standpoint of the method we associate with the early work on integration by Roberval, L'Hôpital's accounts, and so forth: the primitive view of infinitesimals, as Roberval et al. define them, which is the result of the conventional reductionist view, or quasi-reductionist view, prevailing in mathematics and mathematical physics today.[1] Nonetheless, it is not the point of view of the *Monadology*.

For example, the simple demonstration of the fallacy of Euler's argument here, from the standpoint of geometry, to which we can hold Euler accountable, is that it is easily demonstrated, beginning with nothing but the circular action of constructive geometry and hence multiply-connected circular action, that we generate discontinuities, or singularities *out of continuity*. These singularities pertain to the nature of monads, at least in respect to the question of magnitude.

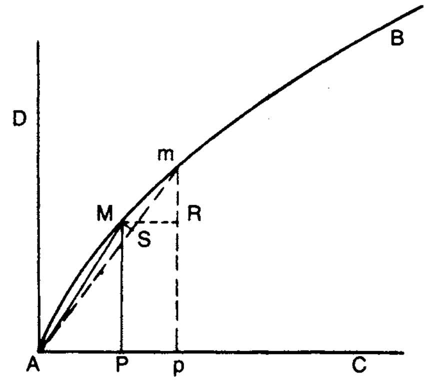
Now, the singularities are not generated by division. They are not generated according to the principle of extension which Euler in these letters demands be the standpoint of examining the *Monadology*. Rather, they are generated with precisely the geometrical qualities which may be attributed to monads by a continuous geometry, which takes no regard of infinitesimals generated by division.

Let us take the case of the simple fallacies which arise from the calculus by the simple method associated with L'Hôpital. If we use L'Hôpital's approach, we cannot equate an infinitesimal to virtually anything; but, in the case where we are trying to get the slope of a discontinuity, this infinitesimal becomes wildly indeterminate in a ponderable degree. That is, the indeterminancy is not infinitesimal, is not marginally infinitesimal, but the indeterminancy is of a very large order of magnitude relative to the function itself. Thus, there is no problem of the type which Euler attributes.

Thus, this is another way of looking at the boundaries of geometrical division, that is, in respect to scale, micro-scale and astrophysical-scale versus the ordinary scale of observation. What we call the micro-scale, the microphysical-scale, or the astrophysical-scale, is associated with the boundary conditions, which are associated in turn with the generation of singularities. What all of this involves, more specifically, is something which is made clearer successively by the work of Leonardo da Vinci, Kepler, Huygens, Leibniz, et al. in the seventeenth century into the eighteenth century.

Huygens, for example, in his treatment of the pendulum clock, shows the role of the cycloid, and of course this extends throughout the entire period, the tautochrone, the isochronic, the brachistochronic, functions, this shows that uni-

FIGURE 1



1. The subject is axiomatics of nonlinearity. I decided to attack some of the problems of conceptual nonlinearity, as against the linear, methods in mathematical physics, from the most elementary, i.e., axiomatic, critical axiomatic standpoint possible. In that respect, some of the sources available through David E. Smith's *A Source Book in Mathematics* (New York: Dover, 1959) and editor Dirk J. Struik's *Source Book in Mathematics: 1200-1800* (Cambridge: Harvard University Press, 1969) are quite useful, as well as some of the other few collateral sources such as Hilbert (see D. Hilbert and S. Cohn-Vossen, *Geometry and the Imagination* [New York: Chelsea, 1952]). I am looking at these, my dear friend Huygens, a few Leibniz things, the Smith and Struik sources, to take some of the most obvious, simple, elementary cases, where the complexities have the greatest relative dependency on the immediate point at issue.

Let's take, just as a point of illustration of what I am doing and what I am thinking about, pages 312 through 316 of Struik, on the L'Hôpital, excerpts.

On pages 312, 313, and 314, we find a development-elaboration of the ground, the basis for two propositions there, and in the following pages, further excerpts from the same source, which give us propositions 163 and 164.

Now if we take that little diagram, as described on pages 313 and 314, pertinent to proposition 1 (**Figure 1**), we have there a simple closed curve, which leads to the proposition that the infinitesimal assumption can be added to make, shall we say, the APM equivalent to *A* small *p*, small *m*, in terms of all the functions associated with that.

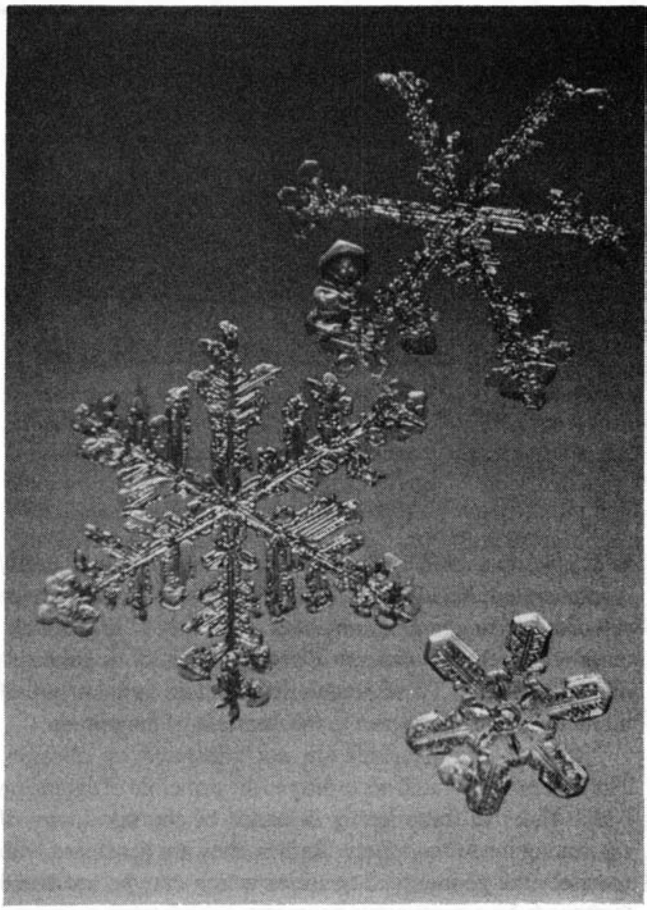
It's very simple to show the fallacy of that. If the curve is not a simple closed, a simple positive curve, but a hyperbola, then we take in the vicinity of the rapidly ascending slope of the hyperbola, we try to make the same construction and that assumption is no longer even approximately true: that, roughly speaking, an apparently infinitesimal difference, even a relatively small difference, is sufficient to throw the whole thing out of whack, and therefore the infinitesimal assumptions cannot be made.

The same thing applies to postulate 2, which begins on the same page, and the same approach applies obviously hereditarily to postulate 163, 164 in the second selection, which Struik cites from that source.

So, although I think, while this is very simple, what we must do for pedagogical purposes, is look back at the axiomatic assumptions, which we have with Roberval. These axiomatic assumptions in Roberval, the same kind of mathematical assumptions, turn up hereditarily in the case of the L'Hôpital reflection on the work of the Bernoullis. This shows up in the problems of Euler.

So that if we look at this problem of infinitesimals, as defined in these two ways, and we find the fallacy of the notion of the infinitesimal, wherever discontinuities are generated, as in a Weierstrass function, or this much simpler case of the simple single hyperbolic application to this first proposition I cited of L'Hôpital.

It's a lot of fun, it's immediately accessible by people. I just throw that in for a suggestion of how we might approach some of these things, from a pedagogical standpoint, and actually get at the deepest, the most elementary, the most simple axiomatic assumptions which cause propositions in physics and as well as mathematics to go awry.



"On the ordinary macroscale of observation, it appears to us that we have two harmonic orderings: one characteristic of living processes, and the other characteristic of non-living processes, as Kepler treats this matter in his essay on The Six-Cornered Snowflake."

versal lawfulness and determination of time with respect to universal lawfulness is determined in respect to these nonalgebraic functions. The implication of that is that the Cartesian notion of extension, of space, time, and matter, does not exist. Rather, that physical space-time, which has a definite curvature, is what *does exist,* and thus the significance of astrophysics and of microphysics and of the boundary conditions which ostensibly or, putatively, or what not, separate the three domains from one another (or, each of the two extreme domains from the domain of simple observation), and involve the generation of singularities.

The other aspect of this which I stated before and must emphasize again: The characteristic of a monad, in Leibniz's setting, and as I have situated it in the previous little oral memorandum on this subject, is that it is a universality; it is the minimum in which is embedded implicity the maximum, or the minimum in which the maximum is implicitly embedded. This relationship of minimum to maximum is demonstrated immediately from the standpoint of the *Parmenides* dialogue, by the demonstration of the negentropic character of the monad. This we know, from the standpoint of human reason, from examining the nature of human reason itself, or its efficient and therefore existent nature. The fact that we are able to change the potential population density of mankind through scientific and technological progress, i.e., through negentropic processes, nonlinear processes of creative discovery, demonstrates that this process of efficiently expressed discovery is existent and is thus *reason.*

Thus, when we look at man as a monad, as embodying *reason* in this efficient existence sense, we thus define a relationship between the mortal individual, a monad, and the universe as a whole and with the Creator—the reflection of the Creator, the *imago viva Dei.* This negentropic monad, us, the creative reason, individual creative reason, becomes the standpoint from which we understand the monads in general. That is Leibniz's point of view.

Letters 13-15

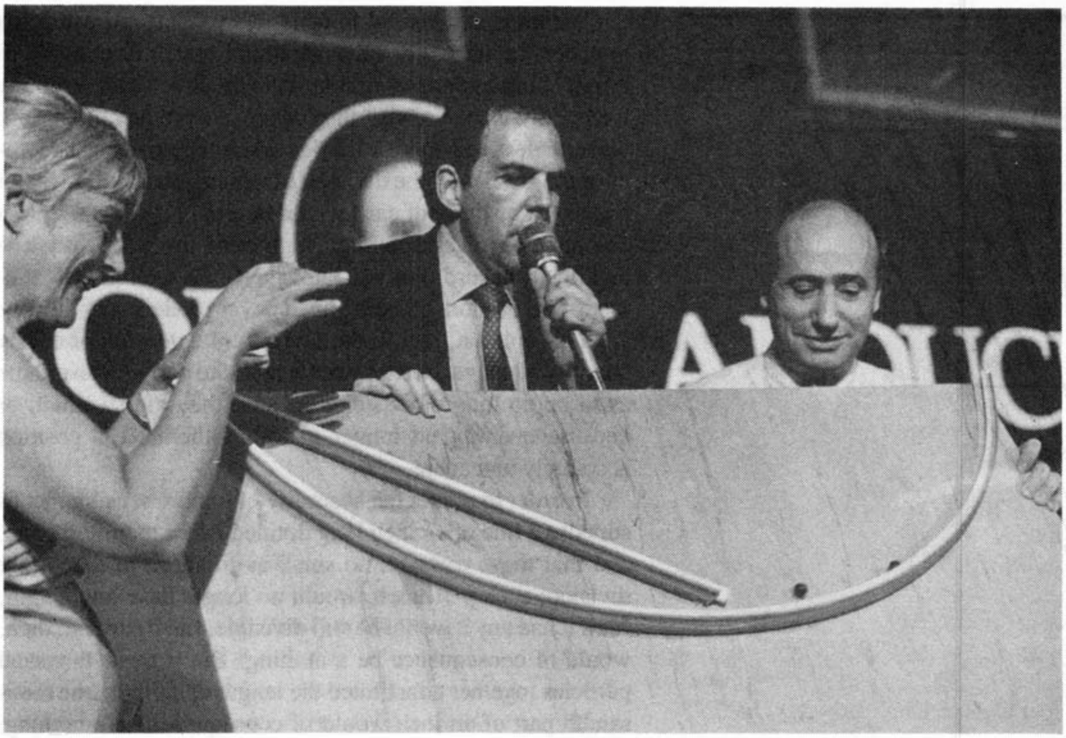
Here we are dealing with Euler's attack on the principle of sufficient reason.

Now, the first thing to look at in Euler's criticism as a whole, particularly when, most to be emphasized when we come to this issue of sufficient reason, is the question of ontology: It is not accidental that Euler starts this entire discussion on extension with the issue of ontology, and affirms infinite divisibility as a corollary of extension to be a quality of substance, a necessary condition, a universal requirement, a universal property, of ontological actuality.

The best vantage point from which to view this, critically, is to recognize the point made by Plato in the *Parmenides* dialogue. Plato anticipates, in effect, this entire argument of Euler's, and of others, by showing through antinomies the inexhaustible absurdity of the idea of simple extension—and does so by showing that simple deductive methods, which are linear methods and hence the method of simple extension, cannot define substance. He does this in the beautiful, ironical method indicated by referencing *change* as the key to the whole business. Thus, not extension, but rather *change in the process of extension,* is the location of efficient ontological actuality.

What Euler does, is to deny the efficiency of monads, except as *deus ex machina*—the Cartesian argument. He says, for example, in this English translation, the Brewster: "In this philosophy everything is spirit, phantom, and illusion; when we cannot comprehend these mysteries, it is our stupidity that keeps up an attachment to the gross notions of the vulgar." And then again, (this as in 14), and in 15, he extends this to include the powers of the soul: that ideational properties are the mechanism which the monadologists profess to be efficient ideas, efficient principles. But, we know precisely that, in respect to change, ideas insofar as they are limited to images of linear space, are not efficient.

So, therefore, by agreeing with Euler on this point, which



A model of the isochronic curvature of the cycloid, demonstrated by Carol White, Mel Klenetsky, and Dino de Paoli at a conference of LaRouche's philosophical associates in September 1990.

he asserts, we thus demolish his argument, because that is not the issue. It is the creative processes through which valid scientific principles are discovered, and changes in human behavior resulting from these ideas, that the monad expresses its efficiency. Therefore, it is not simply an abstract idea of movement, that the idea in this case, that is the creative idea, as distinct from the simple mental image of an object, which is at issue. This, therefore, he assents to, by saying it would be to descend into obscurity to see efficiency in a mere image idea; he avoids the fact that it is not the image idea that is the question here, but as Plato says in the *Parmenides,* it is *change*. The change, in this case, is the change effected by overthrowing an entire set of assumptions controlling human behavior, through discovery of a valid, crucial principle of natural law, and thus changing human behavior to the effect of increasing the per capita power of the human species over the universe.

The sufficient reason in this case applies to the discovery and the elaboration of the discovery of this negentropic characteristic of individual human mortal existence. The fact that human beings have this capability, is sufficient evidence of the existence of this capability within an individual existence within the universe. The fact that this capability within an individual existence expresses a coherence of the maximum and the minimum—that is the maximum in the minimum and the minimum in the maximum—is sufficient to demonstrate, against Euler, that this nature of existence is a general, i.e., maximum, within the universe. General, not in the sense that all existence is immediately manifested, but that it is general in the universe and defines existence.

The *Parmenides* dialogue comes back into play here, by showing the absurdity of any notion of efficient existence from a linear standpoint, the absurdity of the notion of efficient existence from any other standpoint but change.

Selections from Euler's letters

From Letters of Euler on Different Subjects in Natural Philosophy, Addressed to a German Princess, *David Brewster, ed., New York: Harper & Brothers, 1840.*

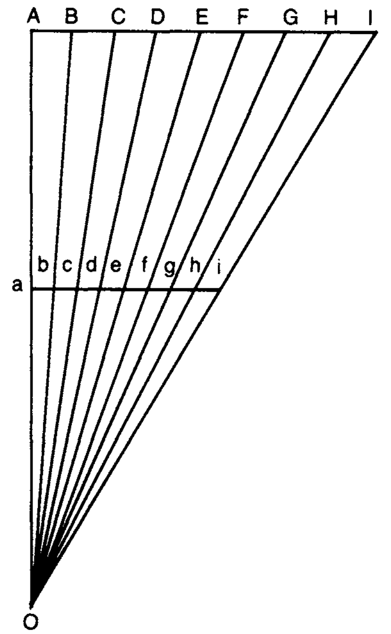
Letter 8: Divisibility of extension *in infinitum*

The controversy between modern philosophers and geometricians, to which I have alluded, turns on the divisibility of body. This property is undoubtedly founded on extension; and it is only in so far as bodies are extended that they are divisible, and capable of being reduced to parts.

You will recollect that in geometry it is always possible to divide a line, however small, into two equal parts. We are likewise by that science instructed in the method of dividing a small line, as *a i,* **Figure 2**, into any number of equal parts at pleasure: and the construction of this division is there demonstrated beyond the possibility of doubting its accuracy.

You have only to draw a line A I parallel to *a i* of any length, and at any distance you please, and to divide it into

FIGURE 2



as many equal parts AB, BC, CD, DE, etc. as the small line given is to have divisions, say eight. Draw afterward, through the extremities A *a*, and I *i*, the straight lines A *a* O, I *i* O, till they meet in the point O; and from O draw towards the points of divisions B, C, D, E, etc. the straight lines OB, OC, OD, OE, etc., which shall likewise divide the small line *a i* into eight equal parts.

This operation may be performed, however small the given line *a i*, and however great the number of parts into which you propose to divide it. It is true that in execution we are not permitted to go too far; the lines which we draw have always some breadth, whereby they are at length confounded, as may be seen in the figure near the point O; but the question is, not what may be possible for us to execute, but what is possible in itself. Now, in geometry lines have no breadth, and consequently can never be confounded. Hence it follows that such division is illimitable.

If it is once admitted that a line may be divided into a thousand parts, by dividing each part into two it will be divisible into two thousand parts, and for the same reason into four thousand, and into eight thousand, without ever arriving at parts indivisible. However small a line may be supposed, it is still divisible into halves, and each half again into two, and each of these again in like manner, and so on to infinity.

What I have said of a line is easily applicable to a surface, and, with greater strength of reasoning, to a solid endowed with three dimensions,—length, breadth, and thickness. Hence it is affirmed that all extension is divisible to infinity; and this property is denominated *divisibility in infinitum.*

Whoever is disposed to deny this property of extension is under the necessity of maintaining that it is possible to arrive at last at parts so minute as to be unsusceptible of any further division, because they cease to have any extension. Nevertheless, all these particles taken together must reproduce the whole, by the division of which you acquired them; and as the quantity of each would be a *nothing* or *cipher* 0, a combination of ciphers would produce quantity, which is manifestly absurd. For you know perfectly well that in arithmetic two or more ciphers joined never produce any thing.

This opinion, that in the division of extension or of any quantity whatever, we may come at last to particles so minute as to be no longer divisible, because they are so small, or because quantity no longer exists, is therefore a position absolutely untenable.

In order to render the absurdity of it more sensible, let us suppose a line of an inch long divided into a thousand parts, and that these parts are so small as to admit of no further division; each part, then, would no longer have any length, for if it had any it would be still divisible. Each particle, then, would of consequence be a nothing. But if these thousand particles together constituted the length of an inch, the thousandth part of an inch would of consequence be a nothing; which is equally absurd with maintaining that the half of any quantity whatever is nothing. And if it be absurd to affirm that the half of any quantity is nothing, it is equally so to affirm that the half of a half, or that the fourth part of the same quantity is nothing; and what must be granted as to the fourth must likewise be granted with respect to the thousandth and the millionth part. Finally, however far you may have already carried in imagination the division of an inch, it is always possible to carry it still further; and never will you be able to carry on your subdivision so far as that the last parts shall be absolutely indivisible. These parts will undoubtedly always become smaller, and their magnitude will approach nearer and nearer to 0, but can never reach it.

The geometrician, therefore, is warranted in affirming that every magnitude is divisible to infinity; and that you cannot proceed so far in your division as that all further division shall be impossible. But it is always necessary to distinguish between what is possible in itself and what we are in a condition to perform. Our execution is indeed extremely limited. After having, for example, divided an inch into a thousand parts, these parts are so small as to escape our sense; and a further division would to us no doubt be impossible.

But you have only to look at this thousandth part of an inch through a good microscope, which magnifies, for example, a thousand times, and each particle will appear as large as an inch to the naked eye; and you will be convinced of the possibility of dividing each of these particles again into a thousand parts: the same reasoning may always be carried forward without limit and without end.

It is therefore an indubitable truth that all magnitude is divisible *in infinitum;* and that this takes place not only with respect to extension, which is the object of geometry, but likewise with respect to every other species of quantity, such as time and number.

28th April, 1761

Letter 10: Of Monads

When we talk in company on philosophical subjects, the conversation usually turns on such articles as have excited violent disputes among philosophers.

The divisibility of body is one of them, respecting which the sentiments of the learned are greatly divided. Some maintain that this divisibility goes on to infinity, without the possibility of ever arriving at particles so small as to be susceptible of no further division. But others insist that this division extends only to a certain point, and that you may come at length to particles so minute that, having no magnitude, they are no longer divisible. These ultimate particles, which enter into the composition of bodies, they denominate *simple beings* and *monads*.

There was a time when the dispute respecting monads employed such general attention, and was conducted with so much warmth, that it forced its way into company of every description, that of the guard-room not excepted. There was scarcely a lady at court who did not take a decided part in favor of monads or against them. In a word, all conversation was engrossed by monads—no other subject could find admission.

The Royal Academy of Berlin took up the controversy, and being accustomed annually to propose a question for discussion, and to bestow a gold medal, of the value of fifty ducats, on the person who, in the judgment of the Academy, has given the most ingenious solution, the question respecting monads was selected for the year 1748. A great variety of essays on the subject were accordingly produced. The president, *Mr. de Maupertuis,* named a committee to examine them, under the direction of the late *Count Dohna,* great chamberlain to the queen; who, being an impartial judge, examined with all imaginable attention the arguments adduced both for and against the existence of monads. Upon the whole, it was found that those which went to the establishment of their existence were so feeble and so chimerical, that they tended to the subversion of all the principles of human knowledge. The question was therefore determined in favor of the opposite opinion, and the prize adjudged to *Mr. Justi,* whose piece was deemed the most complete refutation of the monadists.

You may easily imagine how violently this decision of the Academy must have irritated the partisans of monads, at the head of whom stood the celebrated *Mr. Wolff.* His followers, who were then much more numerous and more formidable than at present, exclaimed in high terms against the partiality and injustice of the Academy; and their chief had well-nigh proceeded to launch the thunder of a philosophical anathema against it. I do not now recollect to whom we are indebted for the care of averting this disaster.

As this controversy has made a great deal of noise, you will not be displeased, undoubtedly, if I dwell a little upon it. The whole is reduced to this simple question, Is a body divisible to infinity? or, in other words, Has the divisibility of bodies any bound, or has it not? I have already remarked as to this, that extension, geometrically considered, is on all hands allowed to be divisible in infinitum; because however small a magnitude may be, it is possible to conceive the half of it, and again the half of that half, and so on to infinity.

This notion of extension is very abstract, as are those of all genera, such as that of man, of horse, of tree, etc., as far as they are not applied to an individual and determinate being. Again, it is the most certain principle of all our knowledge, that whatever can be truly affirmed of the genus must be true of all the individuals comprehended under it. If therefore all bodies are extended, all the properties belonging to extension must belong to each body in particular. Now all bodies are extended, and extension is divisible to infinity; therefore every body must be so likewise. This is a syllogism of the best form; and as the first proposition is indubitable, all that remains is to be assured that the second is true, that is, whether it be true or not that bodies are extended.

The partisans of monads, in maintaining their opinion, are obliged to affirm that bodies are not extended, but have only an appearance of extension. They imagine that by this they have subverted the argument adduced in support of the divisibility *in infinitum*. But if body is not extended, I should be glad to know from whence we derived the idea of extension; for if body is not extended, nothing in the world is, as spirits are still less so. Our idea of extension, therefore, would be altogether imaginary and chimerical.

Geometry would accordingly be a speculation entirely useless and illusory, and never could admit of any application to things really existing. In effect, if no one thing is extended, to what purpose investigate the properties of extension? But as geometry is beyond contradiction one of the most useful of the sciences, its object cannot possibly be a mere chimera.

There is a necessity then of admitting, that the object of geometry is at least the same apparent extension which those philosophers allow to body; but this very object is divisible to infinity: therefore existing beings endowed with this apparent extension must necessarily be extended.

Finally, let those philosophers turn themselves which way soever they will in support of their monads, or those ultimate and minute particles divested of all magnitude, of which, according to them, all bodies are composed, they still plunge into difficulties, out of which they cannot extricate themselves. They are right in saying that it is a proof of dullness to be incapable of relishing their sublime doctrine; it may however be remarked, that here the greatest stupidity is the most successful.

5th May, 1761.

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