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# Brunelleschi and the Quantization of Space

by Lyndon H. LaRouche, Jr.

The one major work of science on which I hope to complete my essential contributions is the establishment of an adequately intelligible representation of the negative curvature of physical space-time in the regions of singularities within a Riemann surface function of otherwise everywhere positive curvature.

On this account, I wish to emphasize our indebtedness to the relevant work of Filippo Brunelleschi. Although I must confess that I do this, in part, out of love for the memory of that great scientist, my principal motive is a broader and more immediately practical one. These remarks are devoted to a brief explanation of that broader purpose.

My strength in these matters originates in a project of philosophical studies begun at the age of 12, which won me forever to the standpoint of Leibniz's *Monadology*, *Theodicy*, and certain other writings by the age of between 13 and 14. All that I have accomplished in relevant matters, is derived from my undertaking, shortly after that, a defense of Leibniz against the arguments of Kant's *Critiques*. My refutation of the central dogmas of Kant, as summarized in his *Critique of Judgment*, became the notion of intelligibility of the creative mental processes from which is derived everything I deem particularly useful in my attempted contributions to human knowledge.

The overriding importance which I attribute to a Socratic treatment of axiomatics, over mere formal consistency, puts me at a distance from prevailing modern ideas about scientific knowledge and much closer to the spirit of the Golden Renaissance. In such matters, that is a weakness in my work, but also an advantage whenever axiomatic issues of fundamentals respecting ontology are the proper point of emphasis, as is the case in this matter of the axiomatic substrate of notions of curvature of physical space-time. My special viewpoint, so identified, is a valuable contribution to the division of labor on the subject of quantization of physical space-time.

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*Multiply-connected physical least action: The familiar Red Spot in Jupiter's atmosphere.*

Properly defined, the "quantization" of physical space-time signifies a rejection of the approach to physical science associated with the neo-Euclidean formalisms of Descartes, Newton, and so on. In the view for

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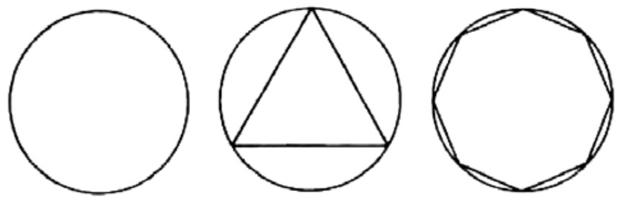
FIGURE 1
### Nicholas of Cusa's Circle



*Nicholas of Cusa, in his 1440 book,* On Learned Ignorance, *showed geometrically that human reason is not attainable through mere logical thought. If we attempt to approach a circle (reason) through construction of polygons with more and more sides (logical thought), it might be thought that we would actually get closer and closer to a circle. Nonsense! A circle has no angles; the more angles we add to the polygon, the further we are from a circle.*

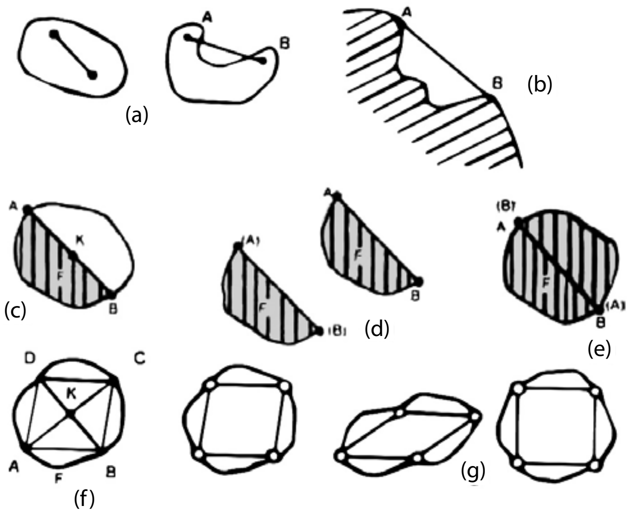
which I speak, no discrete existence of the sort we tend to associate with naive sense certainty is permitted the quality of self-evident existence. Rather, everything which seems to be a discrete existence is something constructed out of what first appears to our imagination as an undifferentiated continuum of a constructive-geometric representation of multiply connected physical least action.

At first, the isoperimetric principle defined by Nicholas of Cusa suggests that the continuum must be defined in terms of multiply connected circular action as the elementary form of physical least action (**Figures 1-2**).

Later, with the work of Gauss, Dirichlet, Riemann, and Weierstrass, we have the higher geometry of the Gauss-Riemann complex domain. This latter domain, in which the characteristic form of functions is associated predominantly with elliptic and hyperbolic trigonometries, is generated by replacing circular with self-similar spiral forms of multiply connected least action (**Figure 3**).

From this more advanced standpoint, the construction of the kinds of singularities associated with electromagnetic generation of discrete existence from continuous least action becomes implicitly susceptible of intelligible representation. To an expanding degree, we are enabled to elaborate viable functional representations for processes, when adequate such representations of nonlinear processes are not possible in any other known way. Additionally, as Riemann indicated in his dissertation on representation of an arbitrary function, it is implicit that all really existing physical processes are susceptible of representation from such a standpoint (see box, page 14).

FIGURE 2
### Least Action: The Isoperimetric Principle




*About 400 years after Cusa, Jacob Steiner devised the following proof that the circle is the figure that encompasses a maximum area for a given perimeter, also without the use of algebraic axioms. If it is assumed that another figure has been discovered that has this property, then this figure must at least be convex; otherwise, a connecting line could always be drawn from A to B that increases the area of the figure and decreases the perimeter (a).*

*Take an arbitrary figure (b). The first step—if it is concave—is to transform it into a convex figure by wrapping a string around the figure. This increases the area by the amount shown but decreases the perimeter. Therefore, the last step here is to expand the figure by a continuous amount along its entire edge to bring the perimeter back to its original length.*

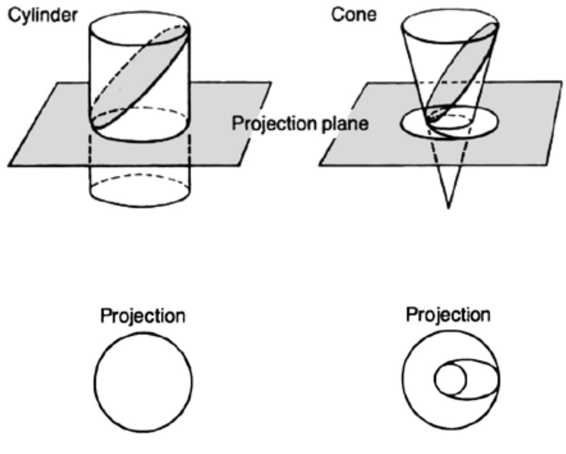
*The second step is to make the figure symmetrical. To do this, divide the perimeter into two parts of equal length, AB and BA (for example by measuring the perimeter with a string and then folding the string in half) (c). Then the figure can be divided along the straight line that joins A and B. Choose the larger of two halves (d). Cut the other half out and rotate the chosen half 180 degrees from A to B (e). Then a symmetrical figure is produced with the perimeter of the original figure and possibly with a greater area. If the new figure is no longer convex, it can be made so by application of the first step.*

*Next, fold the resulting figure in half twice (f) creating the points A, B, C, and D. Join them with straight lines. They will form either a square or a rhombus parallelogram as shown. If it is a square, we are finished and have transformed the figure into a circle. If it is a rhombus, then the area of the figure can be increased by "straightening" the rhombus into a square, while the perimeter does not change (g).*

*If this procedure is repeated, then the figure will get closer and closer to a circle. The circle is the only figure whose area cannot be increased in this way.*

FIGURE 3
### Conical Versus Cylindrical Action

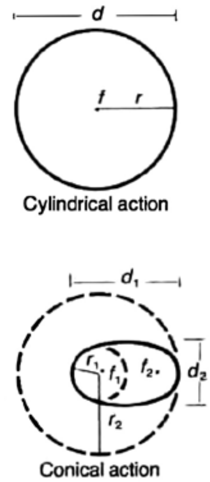
(a) Projection of elliptical cuts through the cylinder and cone



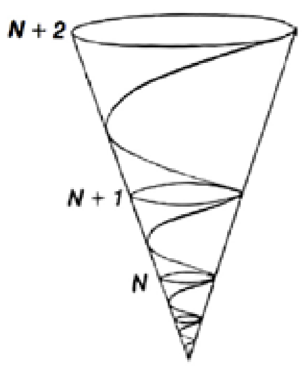

(b) Transformation produced by cylindrical and conical action

**Circle**
Center: $f$
Radius: $r$
Diameter: $d$
Constant curvature

**Ellipse**
Foci: $f_1$, $f_2$
Perihelion: $r_1$
Aphelion: $r_2$
Major axis: $d_1$
Minor axis: $d_2$
Inflection points in maximum and minimum curvature occur at the end points of the major and minor axes




(c) Series of self-similar expanding circles on a cone




*The qualitative difference between cylindrical and conical action is seen in the projections of elliptical cuts through the cylinder and cone (a). The cut through the cylinder projects as a circle; that is, cylindrical action does not transform the universe. The conic section, however, projects as an ellipse, whose perihelion is the radius of the cone's circular cross section at the base of the cut and whose aphelion is the radius of the circular cross section at the top of the cut. The ellipse demonstrates the transformations produced by conical action.*

*The shift from one to the other is characterized by a transformation from one to two Singular characteristics (Singularities) (b). Instead of a center, the ellipse has two foci; instead of every radius being of equal length (as in the circle), the ellipse's radii vary in length with a minimum (perihelion) and maximum (aphelion); instead of one diameter, the ellipse has major and minor axes.*

*A self-similar series of expanding circles (c) represents the Riemannian transformation from N to N + 1.*

## The Importance of Negative Curvature

It is in this setting that the importance of negative curvature confronts us. The relevance of my axiomatic approach and the broader practical importance of reexamining Brunelleschi's work will become clearer as we proceed to treat the significance of negative curvature.

The most important class of physical functions are those we may describe usefully as elementarily nonlinear. By that we ought to mean that the characteristic feature of the function is an implicitly enumerable density of singularities within the scope of some arbitrarily small interval of action of a continuing physical process.

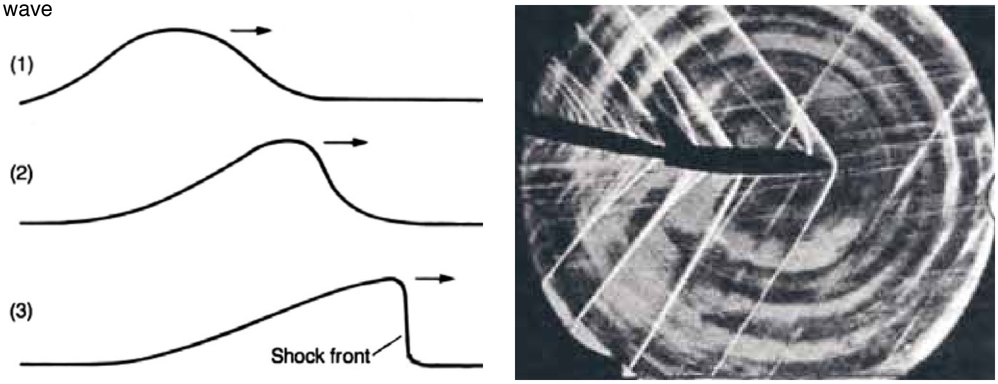
This class of functions is much more than merely very important. All living processes, if adequately represented, are nonlinear processes of this sort. Additionally, at the extremes of scale of astrophysics and microphysics, we are obliged to adduce anything corresponding to an elementary law of nature from nothing but the curvature of physical space-time. Thus, we know that all truly elementary physical functions are of the form of nonlinear propositions within the terms of reference of the Gauss-Riemann complex domain. In the elementary domains of astrophysics, microphysics, and biophysics, no discrete magnitudes exist self-evidently. They exist in the geometric form of construction of singularities from a continuous manifold.

Thus, the derivation of the elementary laws of physics from nothing outside the curvature of physical space-time presents us with a notion of the quantization of physical space-time. This quantization references the generation of discreteness as singularities, and also references the harmonic ordering of variable densities of singularities within a defined interval of action within the continuum. It is only in that sense that I reference the subjects of quantization of space and of nonlinear functions. Up to a point, the Riemann surface function

FIGURE 4
**Shock Waves and Nonlinearity**

(a) Propagation of a nonlinear pressure wave

(b) Formation of sonic boom shock wave




*As a pressure wave travels through air, it takes the shapes in (a). The wave is termed nonlinear because the higher pressure area moves forward faster than the lower pressure area. It tends to lean forward, similar to an ocean wave beginning to break at the beach. In this case, however, the wave forms a sharp front that has a "shock" effect if it hits an obstacle.*

*The formation of a sonic boom wave at the front of a jet plane traveling at supersonic speed is shown in (b). Moving faster than the speed of sound, the jet piles up the air in front of it, creating a pressure situation similar to the shock in (a).*

appears to be an adequate method of representation of nonlinear processes. This function accounts for what must happen in a process to bring about restored connectivity following the earlier appearance of a singularity. However, this representation is an inadequate one, which Riemann's collaborator Beltrami was the first to show in a forceful way.

The problematic issue here is an inadequacy in Dirichlet's topological principle respecting the manner in which connectivity is restored after the generation of a singularity within a Gaussian manifold. It happens that this flaw in the Dirichlet principle is an axiomatic one, and, since the issue of method involved Socratic treatment of axiomatics, the matter is therefore one which is more than merely of great interest to me.

In physics terminology, the Riemann surface function aids us in representing what has happened in the transition from one phase state to the next of a nonlinear

## Riemannian Geometry, Nonlinearity, and Negentropy

Bernhard Riemann's most significant contribution was to prove that the standard mathematical methods used in theoretical physics do not work. Riemann's 1859 paper, "On the Propagation of Plane Air Waves of Finite Amplitude," demonstrates how under certain conditions an intense sinusoidal air wave will change its form as it moves, transforming itself into a shock front across which a discontinuous change of pressure occurs (see **Figure 4**). Up to the point of formation of the shock front, the propagation of the wave appears to be adequately described by the usual differential equations of hydrodynamics. At the formation of the shock front, however, some of the parameters of these equations assume infinite values. The process has assumed new characteristics; a singularity has been formed.

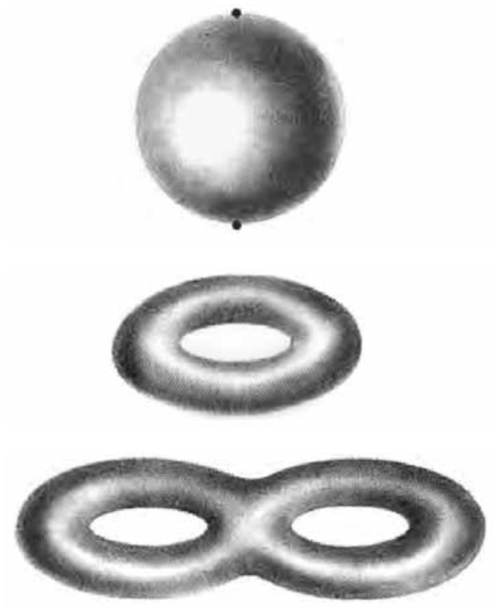
Riemann brings out here the fact that the underlying processes of the universe have the potential to fundamentally change their characteristics of action through the mediation of singularities—what appear in the discrete, visible manifold as "individuals" (a shock wave, for example). At the same time, new potentialities, or degrees of freedom, are opened up for further transformation.

Another example of the same law of the continuous manifold is revealed in the familiar phase changes in matter, like the freezing of water, where the transformation from liquid to solid is accompanied by the appearance of a new singularity-type, the water crystal.

The only admissible basis for geometry is the process by which a manifold of order N is transformed into a manifold of order N + 1. The subject of geometry is not a point, nor a line, nor a surface, nor a solid, but the process of transformation from point, to line, to surface, to solid, and so on. In other words, Riemann saw the proper subject of geometry as negentropy.

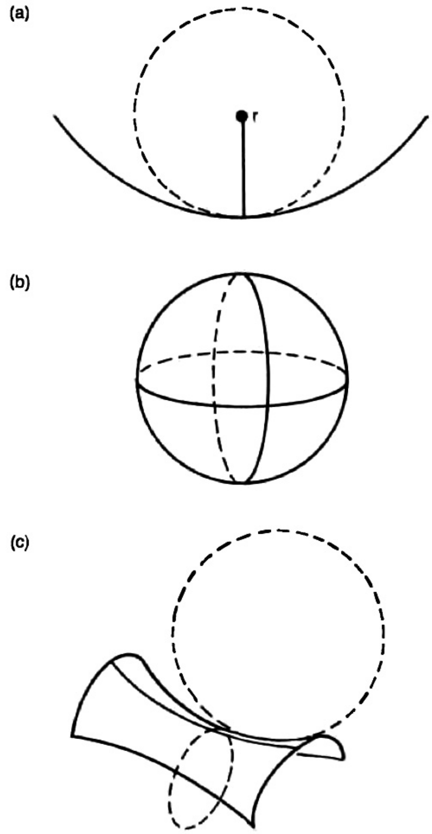
—Dr. Jonathan Tennenbaum

FIGURE 5
**Multiply Connected Surfaces**




*The topology of a sphere has simple connectivity. There are no singularities (holes), only poles. The torus, with its center hole, is doubly connected, and a pretzel shape, with two holes, is triply connected. LaRouche cites Beltrami's work to offer the hypothesis that these singularities are not simply points or holes—empty space—in an otherwise continuous positive curvature, but rather regions whose physical geometry is characterized by negative curvature. Further, LaRouche suggests that these regions correspond to the notion in physics of strong forces.*

FIGURE 6
**Curvature: Negative and Positive**




*Curvature is measured by the radius of a circle that most approximates a curve (a). On a surface, the curvature is measured by two such circles approximating the curvature at the maximum and minimum extremes. These extremes, it turns out, are always perpendicular.*

*The curvature of a surface is positive when these two curves lie on the same side of the surface, as in a sphere (b) or a torus. On a surface of negative curvature the two circles will lie on opposite sides of the surface, as in the saddle curve (c).*

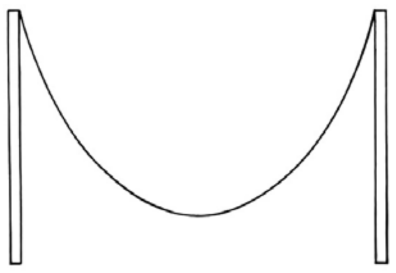
process (**Figure 4**). This representation is true in respect to what we usually reference as weak forces, but is not necessarily true with respect to what we regard relatively as strong forces. That is the physics side of the matter, which I leave to the ministrations of appropriately qualified colleagues. My approach is a more elementary one.

With those limiting considerations, we may say that the Riemann surface function represents what has happened in such cases, but fails to demonstrate how and why that result must occur. What is the causal agency associated with the existence of a topological singularity we represent loosely as a point or hole, which brings about the transformation the Riemann surface function purports to represent after the fact?

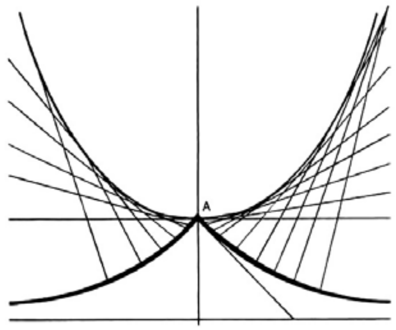
The solution to this problem can lie only within the domain of the constructive geometry of a multiply connected manifold of the Gauss-Riemann sort (**Figures 5-6**). Thus, the kernel of the point: If it is the case, as Beltrami indicates, that these singularities are not simply points or holes in an otherwise continuously positive curvature, but rather regions of negative curvature, and if we were to discover that such regions correspond to the notion of strong forces, we are then on the track of a solution to this interesting problem of axiomatics.

These considerations ought to turn our attention to certain crucial discoveries effected during the 15th century, to matters bearing upon the complementarity of the tractrix and catenary (**Figures 7-8**). It appears not only that Brunelleschi was the first to bring the significance of that to our attention, but that the physics of his design

FIGURE 7
**The Catenary and the Tractrix**



(a) Catenary

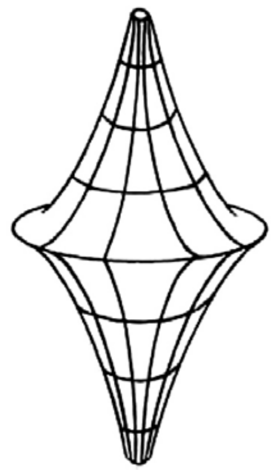



(b) Tractrix (involute of catenary)

*The catenary is the form assumed by a chain or rope suspended from two fixed points and hanging under its own weight (a). The surfaces between the ribs of Brunelleschi's dome are families of catenaries.*

*To find the involute of a catenary (or of any curve), imagine a thread on the surface of the curve, which is then cut and unwound from the lowest point on the curve A to the left and the right. The ends of the thread on a catenary rope trace out the tractrix (heavy line). Each step of the unwinding is like constructing a tangent of the catenary to the tractrix. If the normal (perpendicular) is drawn to the tangent of the tractrix at any point, it can be seen that this normal becomes a tangent to the catenary. Note that all tangents from the inside of the tractrix to its base are equal in length.*

FIGURE 8
**Beltrami and Surfaces of Constant Negative Curvature**



*Eugenio Beltrami, the Italian collaborator of Riemann, explored the properties of a pseudosphere, a figure whose surfaces have constant negative curvature. The pseudosphere is generated by rotating a tractrix.*

for the construction of the dome of the cathedral of Florence embodies the application of certain physics implications of this complementarity to applied physics.

The continuation of this line of inquiry by Leonardo da Vinci, and the outgrowths of that in the later work of such as Kepler, Desargues, Leibniz, and Huygens, and such as Monge, Poncelet, and Gauss later, assist us greatly in viewing the internal history of geometrical thinking in modern science from this standpoint of reference.

I make two general points in conclusion.

First, I emphasize that my refutation of Kant's dogmas, as represented in sundry published locations, defines the kinds of creative mental processes associated with valid fundamental discoveries with a form of nonlinear process in which all the problems I have listed are central features.[1] I have also emphasized, that the rigorous scrutiny of the methods of composition employed in great works of strictly classical art forms represent, from the axiomatic standpoint, directly the same quality and form of creative mental activity we encounter in the case of a valid fundamental discovery in physical science.

## The Process of Scientific Discovery

From this vantage point, and the methodological vantage point of Cusa's *De Docta Ignorantia*, the essence of science is not particular knowledge, which is always historically ephemeral in its authority, but rather the process of perfection of the mental powers developed for the work of scientific discovery. In other words, relative to the notions of finiteness associated with formal analysis of the discrete manifold, the active principle of scientific progress is not deductive, but is a transfinite implicitly representable by the kind of nonlinear process indicated.

Hence, in dealing with the axiomatic issues of science, we must adopt the appropriate historical approach to the internal history of science. We must reexamine the branching points in the internal history of science, at which certain axiomatic sorts of ontological assumptions were adopted, and must reexamine the historical issues so posed in terms of reference to new qualities of experimental evidence presently confronting us.

Thus, by reliving the mental experience associated with the most crucial discoveries of a past reaching not too infrequently into 15th-century Italy, we clear confusion from our minds, and approach present-day questions in a fresh way.

Thus, always, when we honor the best contributions of the past, we strengthen the means for solving important tasks of the present.

1. For example, see Lyndon H. LaRouche, Jr., "Designing Cities in the Age of Mars Colonization," *21st Century Science & Technology*, Vol. 1, Nos. 5-6, Nov.-Dec. 1988, pp. 26-48.